ÁLGEBRA Y TRIGONOMETRÍA

1. Álgebra

- Sean $\frac{a}{b}$ y $\frac{c}{d}$ dos números racionales. Entonces:

  $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ $\quad$ $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$ $\quad$ $\frac{a}{b}\frac{c}{d} = \frac{ac}{bd}$ $\quad$ $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

- Sean $a, b \in \mathbb{R}$ y $n, m \in \mathbb{Z}$, entonces:

  $a^n a^m = a^{n+m}$ $\quad$ $(a^n)^m = a^{nm}$ $\quad$ $\frac{a^n}{a^m} = a^{n-m}$ $\quad$ $a^0 = 1$

  $a^{-n} = \frac{1}{a^n}$ $\quad$ $a^n b^n = (ab)^n$ $\quad$ $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ $\quad$ $a^{n/m} = \sqrt[m]{a^n}$

- Sean $a, b \in \mathbb{R}$ y $n, m \in \mathbb{N}$, entonces:

  $\log_n a = \frac{\log_m a}{\log_m n}$ $\quad$ $\log_e a = \ln a$ $\quad$ $\log_{10} a = \log a$ $\quad$ $\log_n n = 1$

  $\log_n (ab) = \log_n a + \log_n b$ $\quad$ $\log_n \left(\frac{a}{b}\right) = \log_n a - \log_n b$

  $\log_n (b^a) = a \log_n b$ $\quad$ $\log_n a = c \Leftrightarrow a = n^c$

- Sean $a, b, x \in \mathbb{R}$ y $n \in \mathbb{Q}$, entonces:

  $(a \pm b)^2 = a^2 \pm 2ab + b^2$ $\quad$ $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

  $(a \pm b)^n = \sum_{r=0}^{n} (\pm 1)^r \begin{pmatrix} n \\ r \end{pmatrix} a^{n-r} b^r$ $\quad$ $T_{r+1} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} a^{n-r} b^r$

  $a^2 - b^2 = (a-b)(a+b)$ $\quad$ $x^2 + (a+b)x + ab = (x+a)(x+b)$

  $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $\quad$ $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

- Sean $a, b, c \in \mathbb{R}$. Las siguientes desigualdades son equivalentes para los signos $\leq, >, \geq$. Además, $a < b$ si $b - a$ es positivo.

  $a < b \Rightarrow a + c < b + c$ $\quad$ $a < b$ y $c < d \Rightarrow a + c < b + d$

  $a < b \Rightarrow ac < bc$ si $c > 0$ $\quad$ $a < b \Rightarrow ac > bc$ si $c < 0$

  $|a| = a$ si $a \geq 0$ $\quad$ $|a| = -a$ si $a < 0$ $\quad$ $\sqrt{a^2} = |a|$

  $|ab| = |a|\,|b|$ $\quad$ $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $\quad$ $|a+b| \leq |a| + |b|$

2. Trigonometría

- Identidades lineales

$\sin\theta = \frac{co}{H}$ $\cos\theta = \frac{ca}{H}$ $\tan\theta = \frac{co}{ca}$ $\cot\theta = \frac{ca}{co}$

$\sec\theta = \frac{H}{ca}$ $\csc\theta = \frac{H}{co}$ $\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

- Identidades cuadricas

$\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta = 1 - \cos^2\theta$ $\cos^2\theta = 1 - \sin^2\theta$

$\tan^2\theta = \sec^2\theta - 1$ $\cot^2\theta = \csc^2\theta - 1$ $\sec^2\theta = 1 + \tan^2\theta$

$\csc^2\theta = 1 + \cot^2\theta$

- Suma (resta) de ángulos

$\sin(a+b) = \sin a\cos b + \sin b\cos a$ $\sin(a-b) = \sin a\cos b - \sin b\cos a$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$ $\cos(a-b) = \cos a\cos b + \sin a\sin b$

$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$ $\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$

- Ángulos dobles

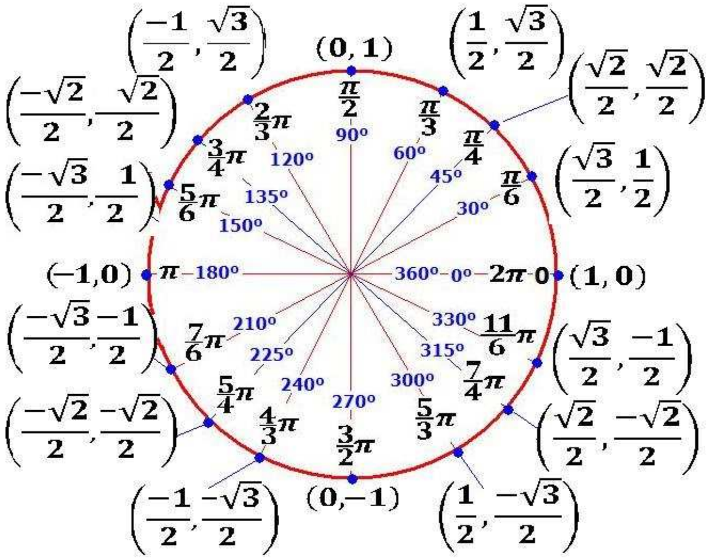
$\sin(2a) = 2\sin a\cos a$ $\cos(2a) = \cos^2 a - \sin^2 a$

$\tan(2a) = \frac{2\tan a}{1 - \tan^2 a}$

- Reducción de Orden

$\sin^2 a = \frac{1 - \cos(2a)}{2}$ $\cos^2 a = \frac{1 + \cos(2a)}{2}$

- Circulo Trigonométrico



- Ley de Senos

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- Ley de Cosenos

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Para las dos leyes, a, b, c representan los lados de un triángulo y A, B, C los ángulos opuestos, respectivamente.